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Radiation damping of a quantum particle with a spin magnetic moment

Anatoly Yu Smirnov

Radiophysics Department, N Lobachevsky State University, Gagarin avenue 23, 603600 Nizhny Novgorod, Russia

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Abstract. It is shown that a quantum non-relativistic particle experiences a radiation friction force due to the interaction of its spin magnetic moment with a blackbody radiation thermal bath. The effect of spin precession in an external magnetic field on deceleration of the particle is considered.

1. Introduction

It is common knowledge that a charged particle is subjected to the force of radiation reaction [1–3]

$$\boldsymbol{F}_{\mathrm{e}} = \frac{2e^2}{3c^3}\frac{\mathrm{d}^3\boldsymbol{r}}{\mathrm{d}t^3}. \tag{1}$$

The quantum version of this force has been obtained in [4, 5]. A neutral non-relativistic particle with a spin magnetic moment $\boldsymbol{\mu}$, say, a neutron, also couples to a quantized electromagnetic field with a vector potential $\boldsymbol{A}(\boldsymbol{r}, t)$. The motion of the particle therewith can be observed in a frame of reference in which the particle's velocity is far less than the velocity of light. Then the interaction of the neutral particle with the electric component $\boldsymbol{E}(\boldsymbol{r}, t) = (-1/c)\dot{\boldsymbol{A}}(\boldsymbol{r}, t)$ of the blackbody radiation field [6]

$$\hat{V}_E = \boldsymbol{\mu}\cdot\left(\frac{\boldsymbol{p}\times\boldsymbol{E}(\boldsymbol{r}, t)}{mc}\right)$$

is much less than the interaction of the magnetic moment $\boldsymbol{\mu} = g\mu_0\boldsymbol{\sigma}$ with the magnetic component $\boldsymbol{B}(\boldsymbol{r}, t) = \operatorname{rot}\boldsymbol{A}(\boldsymbol{r}, t)$:

$$\hat{V}_B = -\boldsymbol{\mu}\cdot\boldsymbol{B}(\boldsymbol{r}, t) = -g\mu_0 L^{-3/2}\sum_k(\boldsymbol{B}_{\boldsymbol{k}}(t)\cdot\boldsymbol{\sigma}(t))\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t)}. \tag{2}$$

As a consequence the neutral non-relativistic particle should experience the magnetic force $\boldsymbol{F}_\mu(t) = \nabla(\boldsymbol{\mu}\cdot\boldsymbol{B}(\boldsymbol{r}, t))$. Here $\boldsymbol{p}$ is the canonical momentum of the particle, $\boldsymbol{\sigma} = \{\sigma_i\}(i = 1, 2, 3)$ is the set of Pauli matrices, L is the linear size of the system, and $\boldsymbol{B}_{\boldsymbol{k}}(t)$ are the space harmonics of the fluctuating magnetic field $\boldsymbol{B}(\boldsymbol{r}, t)$. It should be noted that the forthcoming quantum derivation is restricted to spin-$\frac{1}{2}$ particles, so that all Heisenberg unaveraged variables $\boldsymbol{B}_{\boldsymbol{k}}(t), \boldsymbol{r}(t), \ldots$ are (2×2) matrices. The angular brackets $\langle\ldots\rangle$ will denote thereafter an ensemble average over the initial state of the blackbody radiation heat bath with a temperature T (the Boltzmann constant $k_{\mathrm{B}} = 1$) in combination with the trace over spins. For a neutron the g-factor g and the magneton μ_0 are $g = -1.9$, $\mu_0 = e\hbar/2m_{\mathrm{p}}c$

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with m_{p} being the proton mass, whereas for an electron $g = 1$ and $\mu_0 = e\hbar/2m_0c$ with m_0 being the electron mass.

The aim of this paper is to show that the interaction of a magnetic moment with a blackbody radiation field gives rise to a frictional force $\boldsymbol{F}_\mu(t)$ decelerating the motion of a neutral particle with a spin magnetic moment in a photon thermal bath. The usual radiation reaction force (1) is determined by the uniform self-interaction electric field $\boldsymbol{E}(t)$, whereas the inhomogeneous blackbody radiation magnetic field $\boldsymbol{B}(\boldsymbol{r}, t)$ can only give rise to the magnetic friction. As a result, the force $\boldsymbol{F}_\mu(t)$ is found (see equation (17) later) to have more than three time derivatives provided the magnetic moment is kept constant:

$$\boldsymbol{F}_\mu(t) = -\frac{2(g\mu_0)^2}{3c^5}\frac{\mathrm{d}^5\boldsymbol{r}}{\mathrm{d}t^5}. \tag{3}$$

Generally, both the magnetic radiation reaction and blackbody field fluctuations contribute significantly to the magnetic frictional force. Therefore, the time evolution of the magnetic moment, say, a spin precession under the uniform magnetic field $\boldsymbol{B}_0$ action, may have a pronounced effect on the magnetic frictional force, and with it on the space motion of the particle. Then the magnetic frictional force is found (see equations (18) and (19)) to be proportional to the particle velocity $V_j(t) = \mathrm{d}r_j(t)/\mathrm{d}t$ $(j = 1, 2, 3)$: $(\boldsymbol{F}_\mu)_j = -\zeta_j(\Delta, T)V_j(t)$. A force of this type brings the quantum particle to rest with respect to a frame of blackbody radiation in contrast to the usual radiation reaction force (1).

Here we restrict ourselves to the quantum-mechanical treatment; however, a classical version of the force $\boldsymbol{F}_\mu$ (3) can also be obtained.

Note that the radiation reaction force for classical spinning particles is particularly considered in the work of Barut and Unal [7]. These authors find a correction to the Lorentz–Dirac equation due to Zitterbewegung (the helical motion of the charge around the centre of mass) which models spin; in so doing the third time derivative of the particle position is taken into account at most. In contrast, we examine quantum spin properties of the non-relativistic particle more rigorously and consider in some detail the frequency dependence of the magnetic frictional force. We do not use a classical spin theory as Barut and Unal do, but we are dealing with the agreed-upon representation of spin variables in terms of Pauli matrices. Therefore, for a quantum non-relativistic particle our starting model may be thought to be more correct than that of Barut and Unal, even though the ensuing development of the paper [7] appears sufficiently rigorous.

The existence of the magnetic frictional force may be of importance for the theory of nuclear radiation relaxation [8, 9]. In particular, this force makes a contribution to the broadening of the excited energy levels of a neutron in an atomic nucleus due to the interaction of the spin magnetic moment with the blackbody radiation heat bath. The mechanism of neutron energy dissipation in question also has astrophysical applications, for instance in the theory of neutron stars and pulsars [10].

2. Equations

The Hamiltonian of a quantum non-relativistic particle with zero electric charge, mass m and spin magnetic moment $\boldsymbol{\mu} = g\mu_0\boldsymbol{\sigma}$, coupled to a blackbody radiation thermal bath and subjected to a constant z-axially directed magnetic field $\boldsymbol{B}_0$ can be written in the form

$$\hat{H} = \frac{p^2}{2m} + \hat{V}_B - \boldsymbol{\mu}\cdot\boldsymbol{B}_0 + \hat{H}_B \tag{4}$$

with

$$\hat{H}_B = \sum_{k,s} \hbar\omega_k (a^+_{k,s} a_{k,s} + \tfrac{1}{2})$$

being the Hamiltonian of the free photon heat bath, and $a^+_{k,s}$, $a_{k,s}$ the creation–annihilation operators of photons with a frequency $\omega_k = ck$ and a wavevector $\boldsymbol{k}$.

Fluctuations of a free blackbody radiation field are known to be Gaussian [11]. Therefore, for purposes of calculating the magnetic frictional force $\boldsymbol{F}_\mu$, it is convenient to use the general theory of quantum dynamical systems coupled to a Gaussian thermal bath [12]. Taking into account equation (2) for the interaction $\hat{V}_B$, we find from the Hamiltonian (4) that the Heisenberg operators of the coordinates $r_i(t)$ obey the equations

$$m\ddot{r}_i(t) = g\mu_0 L^{-3/2} \sum_k \mathrm{i}k_i \sigma_j(t)\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t)} B_{\boldsymbol{k},j}(t). \tag{5}$$

The total operator $\boldsymbol{B}_{\boldsymbol{k}}(t)$ of the magnetic field

$$(\boldsymbol{B}_{\boldsymbol{k}}(t))_j = B_{\boldsymbol{k},j}(t) = B^{(0)}_{\boldsymbol{k},j} + L^{-3/2} \int \mathrm{d}t_1 D_{jl}(\boldsymbol{k}, t - t_1) g\mu_0\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t_1)} \sigma_l(t_1) \tag{6}$$

is comprised of unperturbed variables $B^{(0)}_{\boldsymbol{k},j}(t) (j, l = 1, 2, 3)$, descriptive of the free blackbody radiation magnetic field as well as the magnetic field emitted by the spin magnetic moment ('spin light' [13]). Here

$$\begin{aligned} D_{jl}(\boldsymbol{k}, t - t_1) &= \langle\, \mathrm{i}[B^{(0)}_{\boldsymbol{k},j}(t), B^{(0)}_{-\boldsymbol{k},l}(t_1)]_-\rangle\theta(t - t_1) \\ &= 4\pi ck \left(\delta_{jl} - \frac{k_j k_l}{k^2}\right) \sin[ck(t - t_1)]\theta(t - t_1) \end{aligned} \tag{7}$$

is the response function (retarded Green function) of the free quantized magnetic field [11], $\theta(\tau)$ is the Heaviside step function and $\hbar = 1$.

The operators of the dynamical subsystem $\sigma_j(t)\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t)}$ commute with the total Heisenberg operator $B_{\boldsymbol{k},l}(t)$ of the thermal bath. Therefore, after preliminary symmetrization of these factors and averaging of equation (5), we get

$$\begin{aligned} m\langle\ddot{r}_i(t)\rangle = (g\mu_0)^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3}\, \mathrm{i}k_i \int \mathrm{d}t_1 \{&\tilde{M}_{jl}(\boldsymbol{k}, t - t_1)\langle \mathrm{i}[\sigma_j(t)\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t)}, \sigma_l(t_1)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t_1)}]_-\rangle \\ &+ D_{jl}(\boldsymbol{k}, t - t_1)\langle\tfrac{1}{2}[\sigma_j(t)\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t)}, \sigma_l(t_1)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t_1)}]_+\rangle\}. \end{aligned} \tag{8}$$

Here the standard rule of a transition from a sum to an integral $L^{-3}\sum_k \to \int \mathrm{d}^3k/(2\pi)^3$ has been used and the quantum Furutsu–Novikov theorem [12]

$$\begin{aligned} &\langle\tfrac{1}{2}[B_{\boldsymbol{k},j}(t), \sigma_l(t)\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t)}]_+\rangle \\ &\quad = g\mu_0 L^{-3/2} \int \mathrm{d}t_1 \tilde{M}_{js}(\boldsymbol{k}, t - t_1)\langle\, \mathrm{i}[\sigma_l(t)\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t)}, \sigma_s(t_1)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t_1)}]_-\rangle \end{aligned} \tag{9}$$

has been applied. This is possible, because the components $B^{(0)}_{\boldsymbol{k},j}(t)$ of the free blackbody radiation magnetic field are Gaussian with the correlation function [11]

$$\begin{aligned} M_{jl}(\boldsymbol{k}, t - t_1) &= \langle\tfrac{1}{2}[B^{(0)}_{\boldsymbol{k},j}(t), B^{(0)}_{-\boldsymbol{k},l}(t_1)]_-\rangle \\ &= 2\pi ck\left(\delta_{jl} - \frac{k_j k_l}{k^2}\right) \coth\left(\frac{ck}{2T}\right) \cos[ck(t - t_1)] \end{aligned} \tag{10}$$

$$\tilde{M}_{jl}(\boldsymbol{k}, \tau) = M_{jl}(\boldsymbol{k}, \tau)\theta(\tau).$$

3. Results

The dissipative properties of the Brownian particle under study are determined by a sufficiently weak interaction of the magnetic moment with photons having a wavelength $\lambda \sim c/\Delta$, so that $kr \sim r/\lambda \ll 1$. These assumptions let us calculate the commutators $[\sigma_j(t), \sigma_l(t_1)]_\pm$ in the collision terms of equation (8) taking into account the free spin precession [14]

$$\begin{aligned}\sigma_1(t) &= \sigma_1(t_1)\cos\Delta\tau - \sigma_2(t_1)\sin\Delta\tau\\ \sigma_2(t) &= \sigma_1(t_1)\sin\Delta\tau + \sigma_2(t_1)\cos\Delta\tau\\ \sigma_3(t) &= \sigma_3(t_1)\end{aligned}\tag{11}$$

with the frequency $\Delta = 2|g\mu_0 B_0| > 0$ in the constant magnetic field $\boldsymbol{B}_0$ parallel to the z-axis, $\tau = t - t_1$.

It is easy to verify that the averaged longitudinal $\langle\sigma_3(t)\rangle = \sigma_z(t)$ and transversal $\langle\sigma_1(t)\rangle = \langle\sigma_2(t)\rangle = \sigma_{\mathrm{tr}}(t)$ projections of spin matrices relax to the thermodynamic-equilibrium state $\sigma_z^0 = -\tanh(\Delta/2T)$ and $\sigma_{\mathrm{tr}}^0 = 0$, for the sufficiently short time intervals [14, 15]

$$\frac{1}{\tau_z} = \frac{2}{\tau_{\mathrm{tr}}} = \frac{8}{3}(g\mu_0)^2\left(\frac{\Delta}{c}\right)^3\coth\left(\frac{\Delta}{2T}\right).\tag{12}$$

Our prime interest here is the time evolution of the particle space coordinates $r_i(t)(i = 1, 2, 3)$ once the thermodynamic equilibrium in the internal spin space has been established. Taking into account equations (7), (10) and (11) and omitting the brackets $\langle\ldots\rangle$, equation (8) for the averaged displacement $r_i(t)$ may be re-arranged to give

$$\ddot{r}_j(t) + \frac{1}{m}\int \mathrm{d}t_1\tilde{G}_j(t-t_1)[r_j(t) - r_j(t_1)] = 0\tag{13}$$

with $\tilde{G}_j(\tau) = G_j(\tau)\theta(\tau)$ and

$$\begin{aligned}G_j(\tau) = \pi c(g\mu_0)^2\int\frac{\mathrm{d}^3k}{(2\pi)^3}k_j^2k\Bigg\{&\left(1+\frac{k_z^2}{k^2}\right)\Bigg[\left(1-\sigma_z^0\coth\left(\frac{ck}{2T}\right)\right)\\ &\times\sin(\Delta+ck)\tau - \left(1+\sigma_z^0\coth\left(\frac{ck}{2T}\right)\right)\sin(\Delta-ck)\tau\Bigg]\\ &+2\left(1-\frac{k_z^2}{k^2}\right)\sin(ck\tau)\Bigg\}.\end{aligned}\tag{14}$$

It should be emphasized that the displacement $r_i(t)$ in equation (13) is averaged not only over thermal fluctuations, but over the spin variables as well.

In view of translational invariance, the 'collision' term in equation (13) depends solely on the difference between $r_j(t)$ and $r_j(t_1)$. This term involves not only the divergent part (coefficients in front of the even time derivatives of the particle position) but also the finite dissipative components ($\sim \mathrm{d}^3r_j(t)/\mathrm{d}t^3$, etc). Like the Lorentz–Dirac equation [7], the coefficient in front of the second time derivatives leads to mass renormalization, so that the symbol m denotes thereafter the renormalized mass of the particle. Recall that the goal of our work is to study divergence-free dissipative characteristics which are described by the odd part $\tilde{G}(\tau) - \tilde{G}(-\tau) = G(\tau)$ of the function $\tilde{G}(\tau)$. As a consequence the magnetic radiation frictional force $\boldsymbol{F}_\mu(t)$ involved in the relaxation equation

$$m\ddot{r}_j(t) = (\boldsymbol{F}_\mu)_j = \int \mathrm{d}t_1 G_j(t-t_1)r_j(t_1)\tag{15}$$

does not incorporate the Heaviside step function $\theta(\tau)$ and is determined by the Fourier transform $G_j(\omega)$ of the function $G_j(\tau)$:

$$G_j(\omega) = \mathrm{i}\frac{4}{15}\frac{(g\mu_0)^2}{c^5}a_j\left\{\left[1+\sigma_z^0\coth\left(\frac{\Delta+\omega}{2T}\right)\right](\Delta+\omega)^5 \right.$$
$$\left. -\left[1+\sigma_z^0\coth\left(\frac{\Delta-\omega}{2T}\right)\right](\Delta-\omega)^5 + b_j\omega^5\right\} \qquad (16)$$

with the coefficients $a_3 = a_z = 1, b_z = \frac{1}{2}; a_{\mathrm{tr}} = a_1 = a_2 = \frac{3}{4}, b_{\mathrm{tr}} = \frac{4}{3}$. In the absence of a constant magnetic field ($\Delta = 0$) the function $G_j(\omega)$ is $G_z(\omega) = G_{\mathrm{tr}}(\omega) = \mathrm{i}(2/3)(g\mu_0)^2(\omega/c)^5$ to give the equation of motion

$$m\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2} = -\frac{2(g\mu_0)^2}{3c^5}\frac{\mathrm{d}^5\boldsymbol{r}}{\mathrm{d}t^5} = \boldsymbol{F}_\mu \qquad (17)$$

for a quantum neutral particle with a mass m and spin magnetic moment $\boldsymbol{\mu} = g\mu_0\boldsymbol{\sigma}$ in a blackbody radiation field. Similar to the usual 'electric' force $\boldsymbol{F}_{\mathrm{e}}(t)$ (1) of the radiation reaction [1–5], inserting of the magnetic frictional force $\boldsymbol{F}_\mu$ gives rise to runaway solutions of equation (17). It should be stressed that the free blackbody radiation magnetic field $B^{(0)}_{\boldsymbol{k},j}$ does not contribute to the magnetic frictional force (17) if there is no external magnetic field ($B_0 = 0$). In this case the force $\boldsymbol{F}_\mu$ (17) could be derived starting from a spinning particle interacting with the magnetic self-field without introducing any fixed magnetic field. Just as the classical 'electric' force $\boldsymbol{F}_{\mathrm{e}}(t)$ (1) owes its existence to the radiation reaction, so the magnetic frictional force (17) is only due to the magnetic radiation reaction covered by the second term in equation (6). To obtain from equations (2) and (6) the magnetic force $\boldsymbol{F}_\mu(t) = \nabla(\boldsymbol{\mu}\cdot\boldsymbol{B}(\boldsymbol{r},t))$ surviving at $\boldsymbol{B}_0 = 0$ (and $T = 0$) there is a need to replace a spin magnetic moment $g\mu_0\boldsymbol{\sigma}$ by a classical magnetic moment $\boldsymbol{\mu}$ and to average the force $\boldsymbol{F}_\mu(t) = \nabla(\boldsymbol{\mu}\cdot\boldsymbol{B}(\boldsymbol{r},t))$ over an angular distribution of magnetic moments: $\langle\mu_i\mu_j\rangle_\vartheta = \delta_{ij}\mu_0^2$. The factor 2/3 in equation (17) has its origin in this averaging which is equivalent to averaging on the spin variables of the quantum particle.

As is shown in [16], the external magnetic field does not affect the electric radiation frictional force $\boldsymbol{F}_{\mathrm{e}}(t)$. By contrast, the constant magnetic field $\boldsymbol{B}_0$ causes a spin precession with frequency $\Delta = 2|g\mu_0 B_0|$ and thus modifies the interaction between the spin magnetic moment and the blackbody radiation magnetic field. As a result, the function $G_j(\omega)$ (16) is linear in ω at $\omega \ll \Delta$ and non-zero temperatures T of the photon thermal bath. This is to say that the non-relativistic particle with a magnetic moment is subjected to the magnetic frictional force

$$(\boldsymbol{F}_\mu)_j = -\zeta_j(\Delta, T)\frac{\mathrm{d}r_j(t)}{\mathrm{d}t} \qquad (18)$$

which is linearly proportional to the particle velocity, if there is a constant z-axially directed magnetic field $\boldsymbol{B}_0$. Here the temperature-dependent, anisotropic coefficients $\zeta_j(\Delta, T)$ are

$$\zeta_z(\Delta, T) = \frac{8}{15}\frac{(g\mu_0)^2}{T}\left(\frac{\Delta}{c}\right)^5\sinh^{-1}\left(\frac{\Delta}{T}\right) = \frac{4}{3}\zeta_{\mathrm{tr}}(\Delta, T). \qquad (19)$$

For a charged particle with a magnetic moment, such as an electron, the longitudinal mean velocity projection V_z is governed by the relaxation equation

$$\frac{\mathrm{d}V_z}{\mathrm{d}t} - \frac{2\,\mathrm{e}^2}{3mc^3}\frac{\mathrm{d}^2V_z}{\mathrm{d}t^2} + \gamma_z V_z = 0 \qquad (20)$$

with the decrement

$$\gamma_z = \frac{4}{15}\frac{\mathrm{e}^2}{\hbar c}\left(\frac{\Delta}{mc^2}\right)^3\frac{\Delta^2}{\hbar T}[\mathrm{e}^{\Delta/T} - \mathrm{e}^{-\Delta/T}]^{-1}. \qquad (21)$$

This implies that the uniform motion of a spin-$\frac{1}{2}$ particle becomes impossible in the presence of a blackbody radiation heat bath with non-zero temperature, and the magnetic frictional force tends to bring such a quantum particle to rest with respect to a frame of blackbody radiation if there is an external magnetic field.

4. Concluding remarks

In conclusion, it may be said that by virtue of the non-relativistic condition $\Delta \ll mc^2$ the longitudinal damping of electrons in a constant magnetic field is difficult to detect at laboratory test conditions ($T < 10^3$ K, $B_0 < 10^6$ G). However, the results obtained above have astrophysical applications. For instance, a non-relativistic electron moving in the vicinity of a neutron star ($B_0 \sim 10^9$ G , $T \sim 10^6$ K) [1, 10] decelerates for a time $\gamma_z^{-1} \simeq 4.8$ s, whereas the spin relaxation time $\tau_1 \simeq 1.66 \times 10^{-6}$ s and the frequency of spin precession is $\Delta/\hbar = 1.76 \times 10^{16}$ s$^{-1} \ll T/\hbar$. In view of the dependence of the damping rate on the particle mass m, $\gamma_z \sim m^{-7}$, the relaxation of the neutron velocity under the same conditions will be negligibly small. Meanwhile, the magnetic frictional force $\boldsymbol{F}_\mu$ (3) can exert some action on the motion of a neutron in an atomic nucleus resulting in the radiation broadening of excited-state levels. By way of example, magnetic radiation damping of neutron vibrations with the frequency ω_0 is determined by the decrement

$$\gamma_\mu = \frac{2}{3}\left(\frac{g}{2}\right)^2 \frac{\mathrm{e}^2}{\hbar c}\left(\frac{\hbar\omega_0}{mc^2}\right)^3 \omega_0 \tag{22}$$

where g is g-factor of the neutron with a mass m. With the proviso that the energy $\hbar\omega_0$ is of the order of the neutron binding energy in the nucleus ($\hbar\omega_0 \simeq 8$ MeV, $\hbar\omega_0/mc^2 \simeq 0.85 \times 10^{-2}$), the radiation broadening is $\hbar\gamma_\mu \simeq 0.02$ eV, and the lifetime of the neutron excited state is $\tau_\mu \sim \gamma_\mu^{-1} \simeq 2.6 \times 10^{-14}$ s which compares well with the contribution of the quadrupole radiation [8, 9] of the nucleus. Note that the radiation broadening of the energy levels of a charged bound particle will contain contributions both from the magnetic frictional force $\boldsymbol{F}_\mu$ (3) and the electric force $\boldsymbol{F}_{\mathrm{e}}(t)$ (1) with small ratio $\gamma_\mu/\gamma_{\mathrm{e}} = (g/2)^2(\hbar\omega_0/mc^2)^2$.

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